

2019

ECONOMICS — HONOURS

Paper : CC - 4

Full Marks : 65

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Group – A

1. Answer *any ten* questions :

- (a) What are level curves? Give an example of a level curve in economics. 1+1
- (b) Consider the linearly homogeneous production function $Q = f(L, K)$. Using Euler's theorem, show that
- (i) when marginal product of labour is zero, average product of capital equals marginal product of capital, and
- (ii) when marginal product of capital is zero, average product of labour equals marginal product of labour 1+1
- (c) State the duality theorem in the context of linear programming problems. 2
- (d) Consider the function $f(x, y) = (x^2 - y^2)^2$. Is it a homothetic function? Justify your answer. 1+1
- (e) For the following function, show that $f_{23} = f_{32}$ where f_{ij} denotes the second order partial derivative.

$$f(x_1, x_2, x_3) = \left(x_1^2 e^{3x_2 + x_1 x_3} \right) + \frac{2x_2^3}{x_1} \quad 1+1$$

- (f) Consider the Cobb–Douglas production function $y = 50x_1^{1/3}x_2^{2/3}$. Show that the marginal product functions are homogeneous of degree zero. 1+1
- (g) Define saddle point of a function $y = f(x_1, x_2, \dots, x_n)$. 2
- (h) Consider the expenditure function given by $e(p_1, p_2, u_0) = 2(u_0 p_1 p_2)^{1/2}$ where p_1 and p_2 are the prices of the two commodities and u_0 is the target utility level of the consumer. Find the Hicksian or compensated demand functions of the two commodities. 2

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(2)

- (i) Express the following matrix product as a quadratic form and check whether it is positive definite or negative definite. 2

$$[u \ v] \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

- (j) Check whether the following function is concave or convex or neither : 2

$$z = 2x^2 - xy + y^2$$

- (k) Comment on the nature of time path for the following equation : 2

$$y_t = -3 \left(\frac{1}{4} \right)^t + 2$$

- (l) Comment on the quasiconcavity / quasiconvexity of the function $z = x_1^2 + x_2^2$. 2

- (m) What is 'value function' in a constrained optimization problem? Give an economic example of a value function. 1+1

- (n) Use the implicit function theorem to show that $x^2y^3 + 3xy^2 + y = 22$ implies an explicitly defined function $y = f(x)$ at the point (1, 2) and find the value of the derivative $\frac{dy}{dx}$ at this point. 2

- (o) Consider the utility function $u(x_1, x_2) = x_1^2 x_2$. Find the slope of the indifference curve corresponding to this utility function. 2

Group - B

Answer *any three* questions.

2. It is given that $\frac{dy}{dt} = (y - 3)(y - 5)$ where $y \geq 0$.

- (a) Using a phase diagram, show that there are two possible equilibrium levels of y , one at $y = 3$ and the other at $y = 5$.

- (b) Comment on the stability of the equilibria. 2+3

3. Let the utility functions of two individuals L and M be given by –

$$U_L = (X + a)^\alpha (Y + b)^\beta \quad \text{and} \quad U_M = [(X + a)^\alpha (Y + b)^\beta]^2$$

$a, b, \alpha, \beta > 0$. Do you think that the Indifference Curves (ICs) of the two individuals will have the same slope? Why? 1+4

4. State the 'Envelope Theorem'. Explain Roy's identity as an application of the Envelope Theorem. 2+3

5. If $u = Y^{2/3}L^{1/3}$ is the utility function of a person, where Y denotes wage income and L denotes leisure time enjoyed, find out her optimum leisure when wage rate is ₹ 100 per hour. 5

(3)

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6. The profit function of a firm is given by

$$\pi = -R^2 - A^2 + 22R + 18A - 102$$

where A stands for advertisement expenditure and R stands for Research expenditure. Find out the optimum R and A for profit maximisation. Verify the second order condition. 3+2

Group – C

Answer *any three* questions.

7. Consider the following demand and supply functions :

$$Q_d = \alpha - \beta P + \sigma(dP/dt) ; Q_s = -\gamma + \delta P ; (\alpha, \beta, \gamma, \delta > 0)$$

- (a) Assuming that the rate of change of price over time is directly proportional to the excess demand, find the time path $P(t)$.
 (b) What is the intertemporal equilibrium price?
 (c) What restriction on the parameter σ would ensure dynamic stability? 5+2+3

8. (a) A production function is given by $Q = A.L^{\frac{3}{4}} K^{\frac{1}{4}}$.

(i) What is the nature of returns to scale?

(ii) What is the share of K and L in the product, if each factor is paid a price equal to its marginal product? Show that the total product is exhausted.

- (b) Which of the following functions are homothetic? (2+3)+(2½+2½)

(i) $e^{X^2Y} \cdot e^{XY^2}$ (ii) $2 \log X + 3 \log Y$

9. "If $U(x_1, x_2)$ is homogeneous of degree ' r ' in (x_1, x_2) , then $V(p_1, p_2)$ is homogeneous of degree ' $-r$ ' in (p_1, p_2) , where V is the indirect utility function". Prove this result assuming $U = x_1x_2$. 10

10. (a) Solve the following problem graphically :

Maximise : $R = q_1 + 2q_2$

Subject to : $q_1 + q_2 \leq 8$

$2q_1 + q_2 \leq 14$

$q_1, q_2 \geq 0$

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(4)

(b) Consider the following problem :

$$\begin{array}{ll} \text{Maximise} & 3x_1 + 4x_2 \\ \text{Such that} & x_1 + x_2 \leq 10 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1 \leq 8, x_2 \leq 6; x_1, x_2 \geq 0 \end{array}$$

If the optimal solution to its dual problem is $y_1^* = 0$, $y_2^* = 4/3$, $y_3^* = 1/3$, $y_4^* = 0$, find out the optimal solution to its primal problem and verify duality theorem. 5+5

11. Consider the following market model. Find the intertemporal equilibrium price and determine the nature of time path of price. Can you call the equilibrium a stable equilibrium? Illustrate your answer graphically.

$$Q_{dt} = 19 - 6P_t; Q_{st} = 6P_{t-1} - 5; Q_{dt} = Q_{st} \quad 2+3+2+3$$
